HOLES.M

```
## usage: h = holes(y,p,M)
## Function discovered by Edwin A. Suominen
## Written for Octave (GNU MATLAB alternative)

function [h,yi] = holes(y,p,M)

h = zeros(p-M-1,2);

## Compute inverse of y mod p
[d,yi]= gcd(p,y);
if ( yi(2) < 0 )
  yi = p + yi(2);
elseif
  yi = yi(2);
endif

## Compute column 1 of LUT for this key y:
## holes in ascending order
kk = 0; # Counter for iterating next valid hole value
## For all possible hole values...
for i = 1:p-M-1
  ## Compute prospective hole value (may not be valid)
  h(i,1) = M+1 - rem( i*y-(p-M-1) ,p);
  ## If not valid (if >M), set to flag value
  if ( (h(i,1)>M) | (h(i,1)<1) )
    h(i,:) = zeros(1,2);
  else
    kk++; # Increment valid holes counter
  endif
endfor

## Compute column 2 of LUT for this key y: all possible
## overflowing values, xy mod p > M, in ascending order
kk2 = 1; # Counter for iterating next poss. overflow value
## For M+1 (lowest overflow) to p-1 (highest possible)...
for i = M+1:p-1
  ## Compute input value that would produce each
  ## possible overflowing output
  if ( rem(i*yi,p) <= M ) # If input valid...
    h(kk2,2) = i; # ...assign overflow to LUT.
    kk2++; # Move to next available LUT entry
  endif
endfor

## Sort ascending by values in each column
h = sort(h);
if ( 1+(length(h)-kk) > length(h) )
```

```
  h = 0; # If there are no holes (e.g., for y=1)
else
  h = h(1+(length(h)-kk):length(h),:); # Shrink h to omit flag values
endif

endfunction
```

ENCRYPT.M

```
## usage: z = encrypt (x,y,N,k)
## INPUT: input block(s) x, key y, block length N in bits,
## k offset of modulus from 2^N (p=2^N+k)
## OUTPUT: z = x*y mod (2^N+k), but if
## z >= p, z = ( (z-2^N)*y - (k-1) ) mod (2^N+k )
## Function discovered by Edwin A. Suominen
## Written for Octave (GNU MATLAB alternative)

function z = encrypt(x,y,N,k)

L = length(x); # multiple input blocks can be supplied in a vector
z = zeros(L,1); # initialize output vector

## Enforce k must be odd
if ((k/2)==floor(k/2))
  disp('2^N+k cannot be prime is k is even!');
  return;
endif

## Define set order (M) and modulus (p)
M = 2^N; p = M+k;

## Compute LUT of holes in ascending order
## for this key y
h = holes(y,p,M);
Nh = rows(h);

## Basic modulo multipication operation
## Do as array to speed things up
z = rem(x.*y,p);

## For each element in vector...
for i = 1:L

  ## Inventive exception handling
  if (z(i) > M)
    ## Map overflowing value to corresponding hole value in LUT
    ## If there are no holes (h=1 scalar), this code will not
    ## be called because z will always be <= M.
    c = 1:Nh; c = c'; # 1,2 ... (# of valid holes)
    c = c .* ( (z(i)*ones(Nh,1))==h(:,2) ); # Zeros with index of match
    ## z = hole from LUT entry having matching overflow value
    z(i) = h(max(c),1);
  endif

endfor
endfunction
```

DECRYPT.M

```
## usage: x = decrypt (z,y,N,k)
## INPUT: encoded block(s) z, key y, block length N in bits,
## k offset of modulus from 2^N (p=2^N+k)
## OUTPUT: x = z*y^-1 mod (2^N+k), but if
## z = h, where h = ( ((1:k)*y - (k-1) ) mod (2^N+k )
## then z = M+a, where
## a = y^-1 * (2*M+(2+p-h)) mod (2^N+k)
## Function discovered by Edwin A. Suominen
## Written for Octave (GNU MATLAB alternative)

function x = decrypt (z,y,N,k)

L = length(z);  # multiple input blocks can be supplied in a vector

## Enforce k must be odd
if ((k/2)==floor(k/2))
  disp('2^N+k cannot be prime is k is even!');
  return;
endif

## Define set order (M) and modulus (p)
M = 2^N; p = M+k;

## Compute LUT of holes in ascending order
## for this key y
[h,y] = holes(y,p,M); # With two args out, returns y^-1
if ( size(h)==1 )
  Nh = 0; # Account for special case of no holes
else
  Nh = rows(h);
endif

## Done with encryption key y, now y is modulo inverse of orig. y

## For all encrypted blocks (values)...
for i = 1:L

  if Nh>0
    ## If z(i) has been mapped to a hole, restore to overflowing value
    ## For all possible hole values given this key
    for j = 1:Nh
      ## If matches a hole value, remap back
      if (z(i)==h(j,1)), z(i) = h(j,2); endif
    endfor
  endif

endfor
```

```
## Now invert remapped values in vector
## Restored overflowing values will be decrypted properly.
## Do as array to speed things up
x = rem(z.*y,p);      # y = y^1 at this point

endfunction
```

HOLETEST.M

```
## HOLETEST.M
## Written for Octave (GNU MATLAB alternative)

# Np = 1; M = 128; p = M + 3;
Np = 1; M = 512; p = M + 9;

% Try all column (key) values in {1,2,...M}
for j = 2:M,

% Get hole values with brute-force lookup method
x1 = holes1(j,p,M);

% Get hole values using formula discovered by Ed Suominen
x2 = holes2(j,p,M);

disp('');
disp(['j=',num2str(j)]);
disp('  -x1-  -x2-');
disp([x1 x2]);

% Compare
err(j) = sum(abs(x1-x2));
disp(['Sum of absolute differences = ',num2str(err(j))]);

endfor
```

HOLES1.M

```
function h = holes1(y,p,M);

% h = holes1(y,p,N);
% Finds "holes" - skipped values of set {0,1}^N in result
% of x*y mod p. Variable length result with only holes.

% Number of values in set S:{0,1}^N
% M = 2^N;

s = 1:M;    % Working array of values in set S

% Zero out values in set that occur ("non-holes")
  for i = 1:M
    j = rem(i*y,p);  % xy mod p
    % Zero out if not a hole
    if j<=M, s(j) = 0; end
  endfor

% Sort decending to get holes first
h = -sort(-s);
% Trim off zeros (non-holes)
Nnz = sum(h>0); h = h(1:Nnz)';

endfunction
```

HOLES2.M

```
function h = holes2(y,p,M);

% h = holes2(y,p,M)
% Finds "holes" - skipped values of set {0,1}^N in result
% of x*y mod p.
% Uses equation discovered by Edwin A. Suominen

% Number of values in set S:{0,1}^N
% M = 2^N;

k = p-(M+1);

% For vector inputs...
for i=1:length(y)

  for j=1:k,

    ## Input values between M+1 and p will of necessity
    ## be mapped to holes (values not produced by inputs
    ## from set {1,2,...M} because xy mod p is a bijection
    ## (See HAC 1.8 Definition)
    ## h(j,i) = rem( (M+j)*y ,p);
    ## Equation above is simple but doesn't work when
    ## M < xy < p (which happens rarely, but it happens).

    h(j,i) = M+1 - rem(j*y(i)-k,p);
  endfor

endfor

% Map negs. to 0, Sort decending to match formats
Nok = sum(h<=M); h = sort(h); h = h(1:Nok);
if Nok==0, h = []; endif
h = h.*(h>0);
h = -sort(-h);
% Trim off zeros (non-holes)
Nnz = sum(h>0); h = h(1:Nnz);

endfunction
```

1093-093000-1.TXT

```
octave:56> date
ans = [redacted]
octave:57> clock
ans =

  2000.0000          9.0000        30.0000
12.0000     35.0000     32.0890

octave:58> type holes1
holes1  is  the  user-defined  function
defined from: /1093-2/holes1.m

function h = holes1(y,p,M);

% h = holes1(y,p,N);
% CONFIDENTIAL AND PROPRIETARY
% Edwin A. Suominen
% 091600 - Initial writing
% Finds "holes" - skipped values of set
{0,1}^N in result
% of x*y mod p. Variable length result
with only holes.

% Number of values in set S:{0,1}^N
% M = 2^N;

s = 1:M;    % Working array of values in
set S

% Zero  out  values  in  set  that  occur
("non-holes")
  for i = 1:M
    j = rem(i*y,p);  % xy mod p
    % Zero out if not a hole
    if j<=M, s(j) = 0; end
  endfor


% Sort decending to get holes first
h = -sort(-s);
% Trim off zeros (non-holes)
Nnz = sum(h>0); h = h(1:Nnz)';

endfunctionoctave:59> type holes2
holes2  is  the  user-defined  function
defined from: /1093-2/holes2.m

function h = holes2(y,p,M);

% h = holes2(y,p,M)
```

[redacted]

```
% Finds "holes" - skipped values of set
{0,1}^N in result
% of x*y mod p.
% Uses equation discovered by EAS [redacted]

% Number of values in set S:{0,1}^N
% M = 2^N;

k = p-(M+1);

% For vector inputs...
for i=1:length(y)

  for j=1:k,

    ##  Input  values  between  M+1  and  p
will of necessity
    ##  be  mapped  to  holes  (values  not
produced by inputs
    ## from set {1,2,...M} because xy mod
p is a bijection
    ## (See HAC 1.8 Definition)
    h(j,i) = rem( (M+j)*y ,p);

    ##   The   simple   equation   above   is
substituted for the one below
    ## h(j,i) = M+1 - rem(j*y(i)-k,p);
  endfor

endfor

% Map negs. to 0, Sort decending to match
formats
Nok  =  sum(h<=M);  h  =  sort(h);  h  =
h(1:Nok);
if Nok==0, h = []; endif
h = h.*(h>0);
h = -sort(-h);
% Trim off zeros (non-holes)
Nnz = sum(h>0); h = h(1:Nnz);

endfunctionoctave:60> type holetest
holetest  is  the  script  file:  /1093-
2/holetest.m

## HOLETEST.M
##  This  file  is  CONFIDENTIAL  AND
PROPRIETARY.
```

[redacted]

```
##  Written  for  Octave  (GNU  MATLAB
alternative)
## REVISION [redacted]
```

[redacted]

```
Np = 1; M = 512; p = M + 9;

%   Try   all   column   (key)   values   in
{1,2,...M}
for j = 2:M,

% Get hole values with brute-force lookup
method
x1 = holes1(j,p,M);

%   Get   hole   values   using   formula
discovered [redacted]
% by Ed Suominen
x2 = holes2(j,p,M);

disp('');
disp(['j=',num2str(j)]);
disp('  -x1-  -x2-');
disp([x1 x2]);


% Compare
err(j) = sum(abs(x1-x2));
disp(['Sum   of   absolute   differences   =
',num2str(err(j))]);

endforoctave:61> who

*** currently compiled functions:

clock   date    holes1  holes2

octave:62> holetest

j=2
  -x1-  -x2-
  511  511
  509  509
  507  507
  505  505
Sum of absolute differences = 0

j=3
  -x1-  -x2-
  512  512
  509  509
  506  506
  503  503
  500  500
  497  497
Sum of absolute differences = 0

j=4
  -x1-  -x2-
  509  509
  505  505
  501  501
  497  497
  493  493
  489  489
Sum of absolute differences = 0

j=5
  -x1-  -x2-
  511  511
  506  506
  501  501
  496  496
  491  491
  486  486
  481  481
Sum of absolute differences = 0

j=6
  -x1-  -x2-
  509  509
  503  503
  497  497
  491  491
  485  485
  479  479
  473  473
Sum of absolute differences = 0

j=7
  -x1-  -x2-
  507  507
  500  500
  493  493
  486  486
  479  479
  472  472
  465  465
Sum of absolute differences = 0

j=8
  -x1-  -x2-
  505  505
  497  497
  489  489
  481  481
  473  473
  465  465
  457  457
Sum of absolute differences = 0

j=9
  -x1-  -x2-
  512  512
  503  503
  494  494
  485  485
  476  476
```

```
   467  467
   458  458
   449  449
 Sum of absolute differences = 0

 j=10
   -x1-  -x2-
   511  511
   501  501
   491  491
   481  481
   471  471
   461  461
   451  451
   441  441
 Sum of absolute differences = 0

 j=11
   -x1-  -x2-
   510  510
   499  499
   488  488
   477  477
   466  466
   455  455
   444  444
   433  433
 Sum of absolute differences = 0

 j=12
   -x1-  -x2-
   509  509
   497  497
   485  485
   473  473
   461  461
   449  449
   437  437
   425  425
 Sum of absolute differences = 0

 j=13
   -x1-  -x2-
   508  508
   495  495
   482  482
   469  469
   456  456
   443  443
   430  430
   417  417
 Sum of absolute differences = 0

 j=14
   -x1-  -x2-
   507  507
   493  493
   479  479
   465  465
   451  451
   437  437
   423  423
   409  409
 Sum of absolute differences = 0

 j=15
   -x1-  -x2-
   506  506
   491  491
   476  476
   461  461
   446  446
   431  431
   416  416
   401  401
 Sum of absolute differences = 0

 j=16
   -x1-  -x2-
   505  505
   489  489
   473  473
   457  457
   441  441
   425  425
   409  409
   393  393
 Sum of absolute differences = 0

 j=17
   -x1-  -x2-
   504  504
   487  487
   470  470
   453  453
   436  436
   419  419
   402  402
   385  385
 Sum of absolute differences = 0

 j=18
   -x1-  -x2-
   503  503
   485  485
   467  467
   449  449
   431  431
   413  413
   395  395
   377  377
 Sum of absolute differences = 0

 j=19
   -x1-  -x2-
   502  502
```

```
  162  162
  108  108
   54   54
Sum of absolute differences = 0

j=468
  -x1-  -x2-
  424  424
  371  371
  318  318
  265  265
  212  212
  159  159
  106  106
   53   53
Sum of absolute differences = 0

j=469
  -x1-  -x2-
  416  416
  364  364
  312  312
  260  260
  208  208
  156  156
  104  104
   52   52
Sum of absolute differences = 0

j=470
  -x1-  -x2-
  408  408
  357  357
  306  306
  255  255
  204  204
  153  153
  102  102
   51   51
Sum of absolute differences = 0

j=471
  -x1-  -x2-
  400  400
  350  350
  300  300
  250  250
  200  200
  150  150
  100  100
   50   50
Sum of absolute differences = 0

j=472
  -x1-  -x2-
  392  392
  343  343
  294  294
  245  245
  196  196
  147  147
   98   98
   49   49
Sum of absolute differences = 0

j=473
  -x1-  -x2-
  384  384
  336  336
  288  288
  240  240
  192  192
  144  144
   96   96
   48   48
Sum of absolute differences = 0

j=474
  -x1-  -x2-
  376  376
  329  329
  282  282
  235  235
  188  188
  141  141
   94   94
   47   47
Sum of absolute differences = 0

j=475
  -x1-  -x2-
  368  368
  322  322
  276  276
  230  230
  184  184
  138  138
   92   92
   46   46
Sum of absolute differences = 0

j=476
  -x1-  -x2-
  360  360
  315  315
  270  270
  225  225
  180  180
  135  135
   90   90
   45   45
Sum of absolute differences = 0

j=477
  -x1-  -x2-
  352  352
```

```
 308  308
 264  264
 220  220
 176  176
 132  132
  88   88
  44   44
Sum of absolute differences = 0

j=478
 -x1-  -x2-
 344  344
 301  301
 258  258
 215  215
 172  172
 129  129
  86   86
  43   43
Sum of absolute differences = 0

j=479
 -x1-  -x2-
 336  336
 294  294
 252  252
 210  210
 168  168
 126  126
  84   84
  42   42
Sum of absolute differences = 0

j=480
 -x1-  -x2-
 328  328
 287  287
 246  246
 205  205
 164  164
 123  123
  82   82
  41   41
Sum of absolute differences = 0

j=481
 -x1-  -x2-
 320  320
 280  280
 240  240
 200  200
 160  160
 120  120
  80   80
  40   40
Sum of absolute differences = 0

j=482
 -x1-  -x2-
 312  312
 273  273
 234  234
 195  195
 156  156
 117  117
  78   78
  39   39
Sum of absolute differences = 0

j=483
 -x1-  -x2-
 304  304
 266  266
 228  228
 190  190
 152  152
 114  114
  76   76
  38   38
Sum of absolute differences = 0

j=484
 -x1-  -x2-
 296  296
 259  259
 222  222
 185  185
 148  148
 111  111
  74   74
  37   37
Sum of absolute differences = 0

j=485
 -x1-  -x2-
 288  288
 252  252
 216  216
 180  180
 144  144
 108  108
  72   72
  36   36
Sum of absolute differences = 0

j=486
 -x1-  -x2-
 280  280
 245  245
 210  210
 175  175
 140  140
 105  105
  70   70
  35   35
Sum of absolute differences = 0
```

```
j=487
  -x1-  -x2-
  272   272
  238   238
  204   204
  170   170
  136   136
  102   102
   68    68
   34    34
Sum of absolute differences = 0

j=488
  -x1-  -x2-
  264   264
  231   231
  198   198
  165   165
  132   132
   99    99
   66    66
   33    33
Sum of absolute differences = 0

j=489
  -x1-  -x2-
  256   256
  224   224
  192   192
  160   160
  128   128
   96    96
   64    64
   32    32
Sum of absolute differences = 0

j=490
  -x1-  -x2-
  248   248
  217   217
  186   186
  155   155
  124   124
   93    93
   62    62
   31    31
Sum of absolute differences = 0

j=491
  -x1-  -x2-
  240   240
  210   210
  180   180
  150   150
  120   120
   90    90
   60    60
   30    30
Sum of absolute differences = 0

j=492
  -x1-  -x2-
  232   232
  203   203
  174   174
  145   145
  116   116
   87    87
   58    58
   29    29
Sum of absolute differences = 0

j=493
  -x1-  -x2-
  224   224
  196   196
  168   168
  140   140
  112   112
   84    84
   56    56
   28    28
Sum of absolute differences = 0

j=494
  -x1-  -x2-
  216   216
  189   189
  162   162
  135   135
  108   108
   81    81
   54    54
   27    27
Sum of absolute differences = 0

j=495
  -x1-  -x2-
  208   208
  182   182
  156   156
  130   130
  104   104
   78    78
   52    52
   26    26
Sum of absolute differences = 0

j=496
  -x1-  -x2-
  200   200
  175   175
  150   150
  125   125
  100   100
```

```
   75   75
   50   50
   25   25
 Sum of absolute differences = 0

 j=497
   -x1-  -x2-
   192  192
   168  168
   144  144
   120  120
    96   96
    72   72
    48   48
    24   24
 Sum of absolute differences = 0

 j=498
   -x1-  -x2-
   184  184
   161  161
   138  138
   115  115
    92   92
    69   69
    46   46
    23   23
 Sum of absolute differences = 0

 j=499
   -x1-  -x2-
   176  176
   154  154
   132  132
   110  110
    88   88
    66   66
    44   44
    22   22
 Sum of absolute differences = 0

 j=500
   -x1-  -x2-
   168  168
   147  147
   126  126
   105  105
    84   84
    63   63
    42   42
    21   21
 Sum of absolute differences = 0

 j=501
   -x1-  -x2-
   160  160
   140  140
   120  120
   100  100
    80   80
    60   60
    40   40
    20   20
 Sum of absolute differences = 0

 j=502
   -x1-  -x2-
   152  152
   133  133
   114  114
    95   95
    76   76
    57   57
    38   38
    19   19
 Sum of absolute differences = 0

 j=503
   -x1-  -x2-
   144  144
   126  126
   108  108
    90   90
    72   72
    54   54
    36   36
    18   18
 Sum of absolute differences = 0

 j=504
   -x1-  -x2-
   136  136
   119  119
   102  102
    85   85
    68   68
    51   51
    34   34
    17   17
 Sum of absolute differences = 0

 j=505
   -x1-  -x2-
   128  128
   112  112
    96   96
    80   80
    64   64
    48   48
    32   32
    16   16
 Sum of absolute differences = 0

 j=506
   -x1-  -x2-
   120  120
```

```
 105  105
  90   90
  75   75
  60   60
  45   45
  30   30
  15   15
Sum of absolute differences = 0

j=507
 -x1-  -x2-
 112  112
  98   98
  84   84
  70   70
  56   56
  42   42
  28   28
  14   14
Sum of absolute differences = 0

j=508
 -x1-  -x2-
 104  104
  91   91
  78   78
  65   65
  52   52
  39   39
  26   26
  13   13
Sum of absolute differences = 0

j=509
 -x1-  -x2-
 96  96
 84  84
 72  72
 60  60
 48  48
 36  36
 24  24
 12  12
Sum of absolute differences = 0

j=510
 -x1-  -x2-
 88  88
 77  77
 66  66
 55  55
 44  44
 33  33
 22  22
 11  11
Sum of absolute differences = 0

j=511
 -x1-  -x2-
 80  80
 70  70
 60  60
 50  50
 40  40
 30  30
 20  20
 10  10
Sum of absolute differences = 0

j=512
 -x1-  -x2-
 72  72
 63  63
 54  54
 45  45
 36  36
 27  27
 18  18
  9   9
Sum of absolute differences = 0
octave:63> who

*** currently compiled functions:

clock        columns    date          holes1
holes2    num2str  rem       rows

*** local user variables:

M    Np    err  j    p    x1   x2

octave:64> size(M)
ans =

  1  1

octave:65> size(x1)
ans =

  8  1

octave:66> size(err)
ans =

  512    1

octave:67> max(abs(err))
ans = 0
octave:68> 'Simple hole finding function
works!'
ans = Simple hole finding function works!
octave:69> clock
ans =
[redacted]
octave:70> diary off
```

HOLES3.M

```
function [k,h] = holes3(y,p,M);

% h = holes3(y,p,M)
% CONFIDENTIAL AND PROPRIETARY
% Edwin A. Suominen
% Finds "holes" - skipped values of set {0,1}^N in result
% of x*y mod p.
% Uses equation discovered by EAS 9/16/00

% Number of values in set S:{0,1}^N
% M = 2^N;

k = p-(M+1);

% For vector inputs...
for i=1:length(y)

  for j=1:k,

    ## Input values between M+1 and p will of necessity
    ## be mapped to holes (values not produced by inputs
    ## from set {1,2,...M} because xy mod p is a bijection
    ## (See HAC 1.8 Definition)
    ## h(j,i) = rem( (M+j)*y ,p);
    ## Equation above is simple but doesn't work when
    ## M < xy < p (which happens rarely, but it happens).

    h(j,i) = M+1 - rem(j*y(i)-k,p);
  endfor

endfor




if (nargout>=2)
  k = 1:k; k=k';
endif

endfunction
```

TEST3.M

TESTS EACH INPUT FOR ALL KEYS IN SPACE

```
## TEST3.M
## Block size is 10 bits. Input is taken from set Z:{1,2,...1024}
## Because of EAS-invented "pseudogroup" operation, output also
## falls in set Z.
## Keys are also taken from set Z - any set element is OK.

## This test proves the following:
## (1) Output set is same as input set Z.
## (2) Each input value has a unique output value, for a given
## key value.
## (3) The output value from "encrypt.m" can be converted back to
## the input value with "decrypt.m," given the key value.
## (4) For a given input value, each key value produces a unique
## output value.
## Written for Octave (GNU MATLAB alternative)

## No paging - want current screen output
page_screen_output=0;

## Set values defining set and underlying group order
N = 10; M = 2^N; # M = 1024
k = 7; p = M+k; # p = 1031 (prime)

## Create empty matrix of output values
A = zeros(M);

## Define vector with elements of set Z
v = linspace(1,M,M);

## Create string matrix of '-' neutral values for test condition codes
cc = ['-RESULTS- '; ' key: 1234']; # Header
## for each key value...
for i = 1:M
  ## insert key value before neutrals
  ccr = [num2str(i),': ----'];
  ## Leading zeros to make columns line up
  if i<10, ccr = ['0' ccr]; endif
  if i<100, ccr = ['0' ccr]; endif
  if i<1000, ccr = ['0' ccr]; endif
  cc(i+2,:) = ccr;
endfor

############ PART ONE OF TWO ##############
disp(['Tests 1-3, for each key value in set 1,2,...',num2str(M)]);
disp('------------------------------------------------');
```

```
## For all possible key values in Z...
for i = 1:M

  ## Show progress
  disp(['Encrypting and decrypting with key y=',num2str(i),'...']);

  ## Set key value for this iteration
  y = v(i);

  ## Encrypt all possible input values in set Z with key
  b = encrypt(v,y,N,k);
  A(:,i) = b'; # Add this output vector to output matrix

  ## Test for conditions (1),(2) now
  b = sort(b); # Sort ascending

  disp(['Output set: min=',num2str(min(b)),', max=',num2str(max(b))]);


  ##### Test Condition (1) #####
  if ( max(b)==M )
     disp('Output set is same as input set.');
    cc(i+2,7) = '+';
  else
    disp('PROBLEM: Output set larger or smaller than input set!');
    cc(i+2,7) = 'o';
  endif


  ##### Test Condition (2) #####
  ## Each input value should have a unique output value, for a given
  ## key value.
  b = diff(b); # Get differentials between sorted elements
  if ( min(b)==1 & max(b)==1 )
    disp('All elements in output set are unique.');
    cc(i+2,8) = '+';
  else
    disp('PROBLEM: skipped or duplicated element(s) in output set!');
    cc(i+2,8) = 'o';
  endif


  ##### Decrypt output values for this key #####
  b = decrypt(A(:,i),y,N,k)';


  ##### Test Condition (3) #####
```

```
  ## Get differentials between plaintext-encrypted-decrypted (b) and
plaintext (v)
  b = b - v; # Should be all zeros if test passes
  if ( (max(abs(b))==0) )
    disp('All elements in input set encrypt and decrypt with key and
inverse.');
    cc(i+2,9) = '+';
  else
    disp('PROBLEM: One or more elements do not match in
encryption/decryption!');
    cc(i+2,9) = 'o';
  endif

  disp('');

endfor

############ PART TWO OF TWO ##############
disp(['Test 4, for each input value in set 1,2,...',num2str(M)]);
disp('----------------------------------------------------------');

##### Test Condition (4) #####

## For all possible input values in Z, working with full matrix of
outputs
for i = 1:M

  ## Show progress
  disp(['Analyzing outputs for input x=',num2str(i),' with all keys in
set...']);

  ## For a given input value, each key value should produce a unique
output value.
  b = diff(sort(A(i,:))); # Get differentials between sorted elements
  if ( min(b)==1 & max(b)==1 )
    disp('All elements in output set are unique.');
    cc(i+2,10) = '+';
  else
    disp('Skipped or duplicated element(s) in output set.');
    cc(i+2,10) = 'o';
  endif

  disp('');

endfor

## Display test results
disp(cc)
```

RESULTS OF TEST3.M

```
octave:14> date
ans = [redacted]
octave:15> clock
ans =
```

[redacted]

```
octave:16> type test3
test3 is the script file: /1093-2/test3.m

## TEST3.M
## Block size is 10 bits. Input is taken from set
Z:{1,2,...1024}
## Because of EAS-invented "pseudogroup" operation,
output also
## falls in set Z.
## Keys are also taken from set Z - any set element is
OK.

## This test proves the following:
## (1) Output set is same as input set Z.
## (2) Each input value has a unique output value, for a
given
## key value.
## (3) The output value from "encrypt.m" can be converted
back to
## the input value with "decrypt.m," given the key value.
## (4) For a given input value, each key value produces a
unique
## output value.
```

[redacted]

```
## Written for Octave (GNU MATLAB alternative)
```

[redacted]

```
## No paging - want current screen output
page_screen_output=0;

## Set values defining set and underlying group order
N = 10; M = 2^N; # M = 1024
k = 7; p = M+k; # p = 1031 (prime)

## Create empty matrix of output values
A = zeros(M);

## Define vector with elements of set Z
v = linspace(1,M,M);

## Create string matrix of '-' neutral values for test
condition codes
cc = ['-RESULTS- '; ' key: 1234']; # Header
## for each key value...
for i = 1:M
  ## insert key value before neutrals
  ccr = [num2str(i),': ----'];
  ## Leading zeros to make columns line up
  if i<10, ccr = ['0' ccr]; endif
  if i<100, ccr = ['0' ccr]; endif
  if i<1000, ccr = ['0' ccr]; endif
  cc(i+2,:) = ccr;
endfor

############ PART ONE OF TWO ##############
disp(['Tests 1-3, for each key value in s
1,2,...',num2str(M)]);
disp('--------------------------------------------------
----');

## For all possible key values in Z...
for i = 1:M

  ## Show progress
  disp(['Encrypting and decrypting with k
y=',num2str(i),'...']);

  ## Set key value for this iteration
  y = v(i);

  ## Encrypt all possible input values in set Z with key
  b = encrypt(v,y,N,k);
  A(:,i) = b'; # Add this output vector to output matrix

  ## Test for conditions (1),(2) now
  b = sort(b); # Sort ascending

  disp(['Output set: min=',num2str(min(b)),
max=',num2str(max(b))]);

  ##### Test Condition (1) #####
  if ( max(b)==M )
     disp('Output set is same as input set.');
    cc(i+2,7) = '+';
  else
    disp('PROBLEM: Output set larger or smaller th
input set!');
    cc(i+2,7) = 'o';
  endif


  ##### Test Condition (2) #####
  ## Each input value should have a unique output value
for a given
  ## key value.
  b = diff(b); # Get differentials between sorte
elements
  if ( min(b)==1 & max(b)==1 )
    disp('All elements in output set are unique.');
    cc(i+2,8) = '+';
  else
    disp('PROBLEM: skipped or duplicated element(s) i
output set!');
    cc(i+2,8) = 'o';
  endif


  ##### Decrypt output values for this key #####
  b = decrypt(A(:,i),y,N,k)';


  ##### Test Condition (3) #####
  ## Get differentials between plaintext-encrypted
decrypted (b) and plaintext (v)
  b = b - v; # Should be all zeros if test passes
  if ( (max(abs(b))==0) )
    disp('All elements in input set encrypt and decryp
with key and inverse.');
    cc(i+2,9) = '+';
```

```
  else
    disp('PROBLEM: One or more elements do not match in
encryption/decryption!');
    cc(i+2,9) = 'o';
  endif

  disp('');

endfor


############ PART TWO OF TWO ##############
disp(['Test 4, for each input value in set
1,2,...',num2str(M)]);
disp('-----------------------------------------------------
----');

##### Test Condition (4) #####

## For all possible input values in Z, working with full
matrix of outputs
for i = 1:M

  ## Show progress
  disp(['Analyzing outputs for input x=',num2str(i),'
with all keys in set...']);

  ## For a given input value, each key value should
produce a unique output value.
  b = diff(sort(A(i,:))); # Get differentials between
sorted elements
  if ( min(b)==1 & max(b)==1 )
    disp('All elements in output set are unique.');
    cc(i+2,10) = '+';
  else
    disp('Skipped or duplicated element(s) in output
set.');
    cc(i+2,10) = 'o';
  endif

  disp('');

endfor

## Display test results
disp(cc)
octave:18> test3
Tests 1-3, for each key value in set 1,2,...1024
--------------------------------------------------------
Encrypting and decrypting with key y=1...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=2...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=3...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=4...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with k
and inverse.

Encrypting and decrypting with key y=5...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with k
and inverse.

Encrypting and decrypting with key y=6...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with k
and inverse.

Encrypting and decrypting with key y=7...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with k
and inverse.

Encrypting and decrypting with key y=8...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with k
and inverse.

Encrypting and decrypting with key y=9...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with ke
and inverse.

Encrypting and decrypting with key y=10...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with ke
and inverse.

Encrypting and decrypting with key y=11...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with ke
and inverse.

Encrypting and decrypting with key y=12...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with ke
and inverse.

Encrypting and decrypting with key y=13...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with ke
and inverse.

Encrypting and decrypting with key y=14...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with ke
and inverse.
```

```
Encrypting and decrypting with key y=15...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=16...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=17...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=18...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=19...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=20...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=21...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=22...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=23...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=24...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=25...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with k
and inverse.

Encrypting and decrypting with key y=26...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with k
and inverse.

Encrypting and decrypting with key y=27...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with k
and inverse.

Encrypting and decrypting with key y=28...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with ke
and inverse.

Encrypting and decrypting with key y=29...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with ke
and inverse.

Encrypting and decrypting with key y=30...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with ke
and inverse.

Encrypting and decrypting with key y=31...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with ke
and inverse.

Encrypting and decrypting with key y=32...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with ke
and inverse.

Encrypting and decrypting with key y=33...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with ke
and inverse.

Encrypting and decrypting with key y=34...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with ke
and inverse.

Encrypting and decrypting with key y=35...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
```

```
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=1014...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=1015...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=1016...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=1017...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=1018...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=1019...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=1020...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=1021...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=1022...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with key
and inverse.

Encrypting and decrypting with key y=1023...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with k
and inverse.

Encrypting and decrypting with key y=1024...
Output set: min=1, max=1024
Output set is same as input set.
All elements in output set are unique.
All elements in input set encrypt and decrypt with k
and inverse.

Test 4, for each input value in set 1,2,...1024
-----------------------------------------------------------------
Analyzing outputs for input x=1 with all keys in set...
All elements in output set are unique.

Analyzing outputs for input x=2 with all keys in set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=3 with all keys in set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=4 with all keys in set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=5 with all keys in set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=6 with all keys in set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=7 with all keys in set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=8 with all keys in set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=9 with all keys in set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=10 with all keys in set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=11 with all keys in set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=12 with all keys in set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=13 with all keys in set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=14 with all keys in set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=15 with all keys in set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=16 with all keys in set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=17 with all keys in set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=18 with all keys in set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=19 with all keys in set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=20 with all keys in set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=21 with all keys in set...
```

```
Analyzing outputs for input x=995 with all keys in set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=996 with all keys in set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=997 with all keys in set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=998 with all keys in set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=999 with all keys in set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=1000 with all keys in
set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=1001 with all keys in
set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=1002 with all keys in
set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=1003 with all keys in
set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=1004 with all keys in
set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=1005 with all keys in
set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=1006 with all keys in
set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=1007 with all keys in
set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=1008 with all keys in
set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=1009 with all keys in
set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=1010 with all keys in
set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=1011 with all keys in
set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=1012 with all keys in
set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=1013 with all keys in
set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=1014 with all keys
set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=1015 with all keys
set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=1016 with all keys
set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=1017 with all keys
set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=1018 with all keys
set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=1019 with all keys
set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=1020 with all keys
set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=1021 with all keys
set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=1022 with all keys
set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=1023 with all keys
set...
Skipped or duplicated element(s) in output set.

Analyzing outputs for input x=1024 with all keys
set...
Skipped or duplicated element(s) in output set.

-RESULTS-
 key: 1234
0001: ++++
0002: +++o
0003: +++o
0004: +++o
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0009: +++o
0010: +++o
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0027: +++o
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0028: +++o
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1005: +++o
1006: +++o
1007: +++o
1008: +++o
1009: +++o
1010: +++o
1011: +++o
1012: +++o
1013: +++o
1014: +++o
1015: +++o
1016: +++o
1017: +++o
1018: +++o
1019: +++o
1020: +++o
1021: +++o
1022: +++o
1023: +++o
1024: +++o
octave:19> diary off
```

RESULTS OF TEST3B.M

```
octave:4> date
ans = [redacted]
octave:5> clock
ans =

[redacted]

octave:6> type test3b
test3b is the script file: /1093-2/test3b.m

## TEST3B.M
## Block size is 10 bits. Input is taken from set Z:{1,2,...1024}
## Because of EAS-invented "pseudogroup" operation, output also
## falls in set Z.
## Keys are also taken from set Z - any set element is OK.

## This test analyzes outputs for a given input over all
## possible keys.

[redacted]
## Written for Octave (GNU MATLAB alternative)
[redacted]

## No paging - want current screen output
page_screen_output=0;

## Set values defining set and underlying group order
N = 10; M = 2^N; # M = 1024
k = 7; p = M+k; # p = 1031 (prime)

## Define vector with elements of set Z
v = linspace(1,M,M);

## Define vector of skip/repeat counts
cc = zeros(1,M);

disp(['Test for each input value in set 1,2,...',num2str(M)]);
disp('--------------------------------------------------');

## For all possible input values in Z...
for i = 1:M

  ## Show progress
  disp(['Encrypting with input value y=',num2str(i),'...']);

  ## Set input value for this iteration
  x = v(i);

  ## Encrypt input value with all keys in set Z
  for j = 1:M
    b(j) = encrypt(x,v(j),N,k);
  endfor
```

```
  disp(['Output set: min=',num2str(min(b)),', max=',num2str(max(b))]);
  disp('');

  ## Identify any skipped or repeated set elements
  ## with vector of index numbers
  b1 = sort(b); # Sort ascending
  b2 = [diff(b1)']; # Should be all 1's...
  b2 = b2~=1; # ...so 1's indicate skips/repeats

  Nsr = sum(b2); # Count of skips/repeats

  b3 = b2 .* v(1:M-1); # map index numbers to skips/repeats
  b3 = sort(b3); # Sort ascending
  b4 = b3(M-Nsr:M-1); # Select only skips/repeats

  if (Nsr > 0)

    disp(['There are ',num2str(Nsr),' skips & repeats, at:');
    disp(b4);
    disp('----------------------------------------------------------');

    c = zeros(6,Nsr); # Start with empty ("0") matrix
    for j = 1:Nsr
      k1 = max([1 b4(j)-2]);
      k2 = min([b4(j)+3 M]);
      c(1:k2-k1+1,j) = b1(k1:k2);
    endfor

    disp(c)

  endif

  cc(i) = Nsr; # Add this count to vector
  disp(['Maximum skips & repeats for a given input (so far):',num2str(max(cc))]);
  disp('');

endfor

. . .

octave:9> test3b
Test for each input value in set 1,2,...1024
-----------------------------------------------------
Encrypting with input value y=1...
Output set: min=1, max=1024

Maximum skips & repeats for a given input (so far):0

Encrypting with input value y=2...
Output set: min=1, max=1024

There are 6 skips & repeats, at:
     10   518   520  1021  1022  1023
-----------------------------------------------------
      8   515   517  1016  1017  1018
      9   516   517  1017  1018  1020
     10   517   518  1018  1020  1022
     10   517   518  1020  1022  1024
```

```
     11   518   519  1022  1024      0
     12   518   520  1024     0      0
Maximum skips & repeats for a given input (so far):6

Encrypting with input value y=3...
Output set: min=1, max=1024

There are 8 skips & repeats, at:
     10   346   690   692  1016  1018  1020  1022
------------------------------------------------------------
      8   343   686   688  1010  1012  1015  1018
      9   344   687   688  1011  1014  1017  1020
     10   345   688   689  1012  1015  1018  1021
     10   345   688   689  1014  1017  1020  1023
     11   346   689   690  1015  1018  1021  1024
     12   347   689   691  1017  1020  1023     0
Maximum skips & repeats for a given input (so far):8

Encrypting with input value y=4...
Output set: min=1, max=1024

There are 10 skips & repeats, at:
      3   260   519   523   778  1011  1014  1017  1020  1023
------------------------------------------------------------
      1   257   515   518   772  1004  1008  1012  1016  1020
      2   258   516   519   773  1005  1009  1013  1017  1021
      3   259   517   520   774  1006  1010  1014  1018  1022
      3   259   517   520   774  1008  1012  1016  1020  1024
      4   260   518   521   775  1009  1013  1017  1021     0
      5   261   519   522   776  1010  1014  1018  1022     0
Maximum skips & repeats for a given input (so far):10

Encrypting with input value y=5...
Output set: min=1, max=1024

There are 10 skips & repeats, at:
      6   212   418   624   830  1005  1009  1013  1017  1021
------------------------------------------------------------
      4   209   414   619   824   998  1003  1008  1013  1018
      5   210   415   620   825   999  1004  1009  1014  1019
      6   211   416   621   826  1000  1005  1010  1015  1020
      6   211   416   621   826  1002  1007  1012  1017  1022
      7   212   417   622   827  1003  1008  1013  1018  1023
      8   213   418   623   828  1004  1009  1014  1019  1024
Maximum skips & repeats for a given input (so far):10

Encrypting with input value y=6...
Output set: min=1, max=1024

There are 10 skips & repeats, at:
    176   346   518   691   864   999  1004  1009  1014  1019
------------------------------------------------------------
    174   343   514   686   858   992   998  1004  1010  1016
    175   344   515   687   859   993   999  1005  1011  1017
    176   345   516   688   860   994  1000  1006  1012  1018
    176   345   516   688   860   996  1002  1008  1014  1020
    177   346   517   689   861   997  1003  1009  1015  1021
    178   347   518   690   862   998  1004  1010  1016  1022
```

Maximum skips & repeats for a given input (so far):10

Encrypting with input value y=7...
Output set: min=1, max=1023

There are 11 skips & repeats, at:

149	297	445	593	741	889	994	1000	1006	1012	1018
147	294	441	588	735	882	986	993	1000	1007	1014
148	295	442	589	736	883	987	994	1001	1008	1015
149	296	443	590	737	884	988	995	1002	1009	1016
149	296	443	590	737	884	990	997	1004	1011	1018
150	297	444	591	738	885	991	998	1005	1012	1019
151	298	445	592	739	886	992	999	1006	1013	1020

Maximum skips & repeats for a given input (so far):11

Encrypting with input value y=8...
Output set: min=1, max=1024

There are 12 skips & repeats, at:

3	4	261	520	521	779	988	995	1002	1009	1016	1023
1	2	257	515	516	772	980	988	996	1004	1012	1020
2	3	258	516	517	773	981	989	997	1005	1013	1021
3	3	259	517	517	774	982	990	998	1006	1014	1022
3	3	259	517	517	774	984	992	1000	1008	1016	1024
3	4	260	517	518	775	985	993	1001	1009	1017	0
4	5	261	518	519	776	986	994	1002	1010	1018	0

Maximum skips & repeats for a given input (so far):12

Encrypting with input value y=9...
Output set: min=1, max=1024

There are 12 skips & repeats, at:

2	346	347	464	692	922	982	990	998	1006	1014	1022
1	343	344	459	686	915	974	983	992	1001	1010	1019
2	344	345	460	687	916	975	984	993	1002	1011	1020
2	345	345	461	688	917	976	985	994	1003	1012	1021
3	345	345	461	688	917	978	987	996	1005	1014	1023
4	345	346	462	689	918	979	988	997	1006	1015	1024
0	346	347	463	690	919	980	989	998	1007	1016	0

Maximum skips & repeats for a given input (so far):12

Encrypting with input value y=10...
Output set: min=1, max=1024

There are 12 skips & repeats, at:

2	4	416	520	829	933	976	985	994	1003	1012	1021
1	2	412	515	823	926	968	978	988	998	1008	1018
2	2	413	516	824	927	969	979	989	999	1009	1019
2	3	414	517	825	928	970	980	990	1000	1010	1020
3	3	414	517	825	928	972	982	992	1002	1012	1022
3	4	415	518	826	929	973	983	993	1003	1013	1023
0	5	416	519	827	930	974	984	994	1004	1014	1024

Maximum skips & repeats for a given input (so far):12

```
   17   35   53   71   89  107  114  342  458  687  687  859
   19   37   55   73   91  109  114  342  458  687  687  859
   20   38   56   74   92  110  115  343  459  687  688  860
   21   39   57   75   93  111  116  344  460  688  689  861
Maximum skips & repeats for a given input (so far):12

Encrypting with input value y=1014...
Output set: min=1, max=1024

There are 12 skips & repeats, at:
   16   32   48   64   80   96  236  479  661  845  907  969
-------------------------------------------------------------
   14   31   48   65   82   99  240  482  663  846  907  968
   15   32   49   66   83  100  241  483  664  847  908  969
   16   33   50   67   84  101  242  484  665  848  909  970
   18   35   52   69   86  103  242  484  665  848  909  970
   19   36   53   70   87  104  243  485  666  849  910  971
   20   37   54   71   88  105  244  486  667  850  911  972
Maximum skips & repeats for a given input (so far):12

Encrypting with input value y=1015...
Output set: min=1, max=1024

There are 12 skips & repeats, at:
   15   30   45   60   75   90  121  188  511  641  771  901
-------------------------------------------------------------
   13   29   45   61   77   93  125  191  513  642  771  900
   14   30   46   62   78   94  126  192  514  643  772  901
   15   31   47   63   79   95  127  193  515  644  773  902
   17   33   49   65   81   97  127  193  515  644  773  902
   18   34   50   66   82   98  128  194  516  645  774  903
   19   35   51   67   83   99  129  195  517  646  775  904
Maximum skips & repeats for a given input (so far):12

Encrypting with input value y=1016...
Output set: min=1, max=1024

There are 12 skips & repeats, at:
   14   28   42   56   70   84  199  338  614  752  891  961
-------------------------------------------------------------
   12   27   42   57   72   87  203  341  616  753  891  960
   13   28   43   58   73   88  204  342  617  754  892  961
   14   29   44   59   74   89  205  343  618  755  893  962
   16   31   46   61   76   91  205  343  618  755  893  962
   17   32   47   62   77   92  206  344  619  756  894  963
   18   33   48   63   78   93  207  345  620  757  895  964
Maximum skips & repeats for a given input (so far):12

Encrypting with input value y=1017...
Output set: min=1, max=1024

There are 12 skips & repeats, at:
   13   26   39   52   65   78  140  436  437  658  734  956
-------------------------------------------------------------
   11   25   39   53   67   81  144  439  440  659  734  955
   12   26   40   54   68   82  145  440  441  660  735  956
   13   27   41   55   69   83  146  441  441  661  736  957
   15   29   43   57   71   85  146  441  441  661  736  957
```

16	30	44	58	72	86	147	441	442	662	737	958
17	31	45	59	73	87	148	442	443	663	738	959

Maximum skips & repeats for a given input (so far):12

Encrypting with input value y=1018...
Output set: min=1, max=1024

There are 12 skips & repeats, at:

12	24	36	48	60	72	73	231	233	631	632	871
10	23	36	49	62	75	76	234	236	632	633	870
11	24	37	50	63	76	77	235	236	633	634	871
12	25	38	51	64	77	77	236	237	634	634	872
14	27	40	53	66	77	79	236	237	634	634	872
15	28	41	54	67	79	80	237	238	634	635	873
16	29	42	55	68	80	81	237	239	635	636	874

Maximum skips & repeats for a given input (so far):12

Encrypting with input value y=1019...
Output set: min=1, max=1024

There are 12 skips & repeats, at:

11	22	33	44	55	66	336	508	682	769	857	944
9	21	33	45	57	69	340	511	684	770	857	943
10	22	34	46	58	70	341	512	685	771	858	944
11	23	35	47	59	71	342	513	686	772	859	945
13	25	37	49	61	73	342	513	686	772	859	945
14	26	38	50	62	74	343	514	687	773	860	946
15	27	39	51	63	75	344	515	688	774	861	947

Maximum skips & repeats for a given input (so far):12

Encrypting with input value y=1020...
Output set: min=1, max=1024

There are 12 skips & repeats, at:

10	20	30	40	50	60	85	182	275	652	747	936
8	19	30	41	52	63	89	185	277	653	747	935
9	20	31	42	53	64	90	186	278	654	748	936
10	21	32	43	54	65	91	187	279	655	749	937
12	23	34	45	56	67	91	187	279	655	749	937
13	24	35	46	57	68	92	188	280	656	750	938
14	25	36	47	58	69	93	189	281	657	751	939

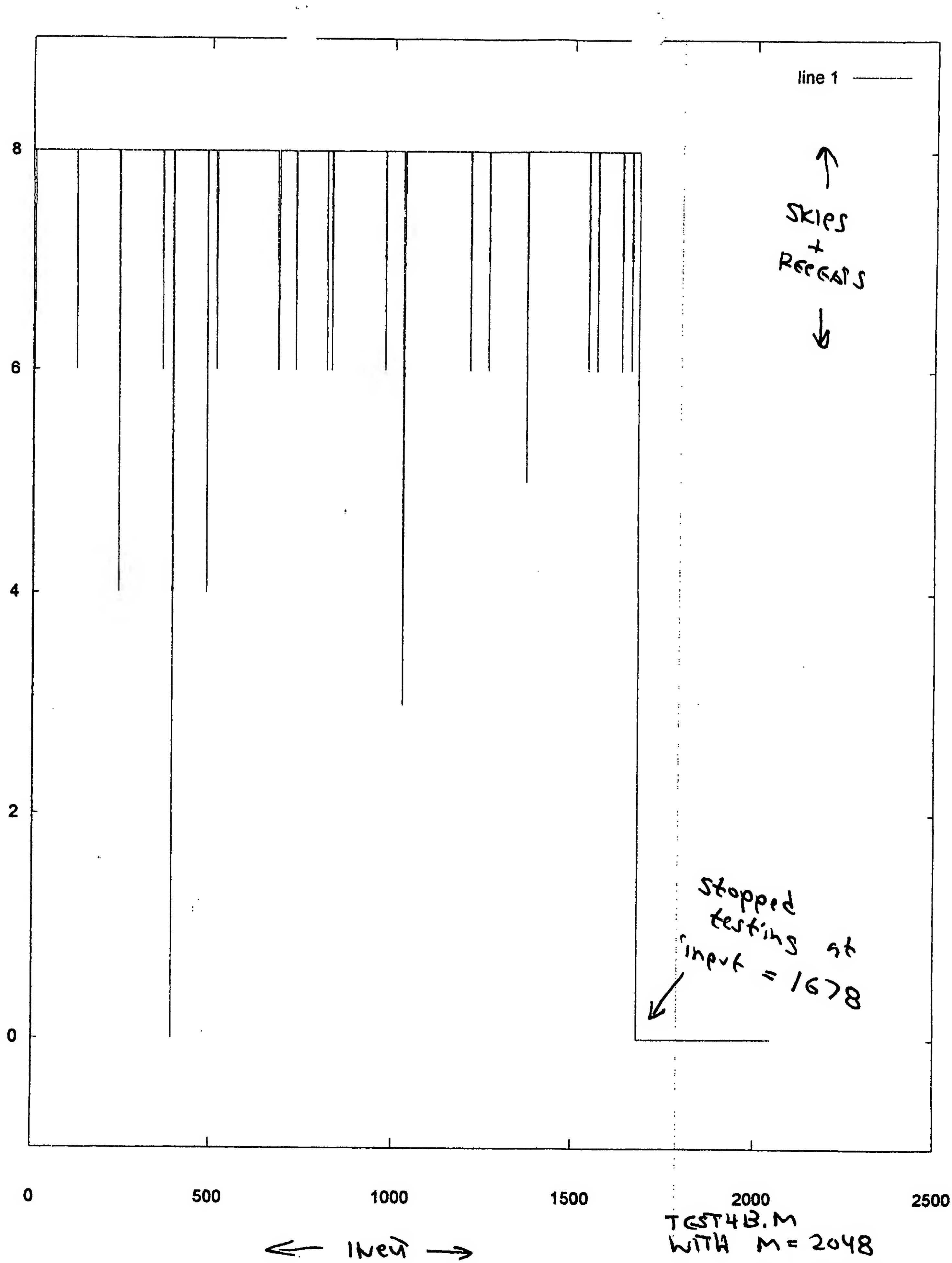
Maximum skips & repeats for a given input (so far):12

Encrypting with input value y=1021...
Output set: min=1, max=1024

There are 12 skips & repeats, at:

9	18	27	36	45	54	199	407	510	615	616	823
7	17	27	37	47	57	203	410	512	616	617	822
8	18	28	38	48	58	204	411	513	617	618	823
9	19	29	39	49	59	205	412	514	618	618	824
11	21	31	41	51	61	205	412	514	618	618	824
12	22	32	42	52	62	206	413	515	618	619	825
13	23	33	43	53	63	207	414	516	619	620	826

```
Maximum skips & repeats for a given input (so far):12

Encrypting with input value y=1022...
Output set: min=1, max=1024

There are 12 skips & repeats, at:
    8   16   24   32   40   48  222  336  567  684  799  915
-----------------------------------------------------------
    6   15   24   33   42   51  226  339  569  685  799  914
    7   16   25   34   43   52  227  340  570  686  800  915
    8   17   26   35   44   53  228  341  571  687  801  916
   10   19   28   37   46   55  228  341  571  687  801  916
   11   20   29   38   47   56  229  342  572  688  802  917
   12   21   30   39   48   57  230  343  573  689  803  918
Maximum skips & repeats for a given input (so far):12

Encrypting with input value y=1023...
Output set: min=1, max=1024

There are 12 skips & repeats, at:
    7   14   21   28   35   42  248  249  511  641  771  901
-----------------------------------------------------------
    5   13   21   29   37   45  252  253  513  642  771  900
    6   14   22   30   38   46  253  254  514  643  772  901
    7   15   23   31   39   47  254  254  515  644  773  902
    9   17   25   33   41   49  254  254  515  644  773  902
   10   18   26   34   42   50  254  255  516  645  774  903
   11   19   27   35   43   51  255  256  517  646  775  904
Maximum skips & repeats for a given input (so far):12

Encrypting with input value y=1024...
Output set: min=1, max=1024

There are 12 skips & repeats, at:
    6   12   18   24   30   36  137  286  435  584  733  882
-----------------------------------------------------------
    4   11   18   25   32   39  141  289  437  585  733  881
    5   12   19   26   33   40  142  290  438  586  734  882
    6   13   20   27   34   41  143  291  439  587  735  883
    8   15   22   29   36   43  143  291  439  587  735  883
    9   16   23   30   37   44  144  292  440  588  736  884
   10   17   24   31   38   45  145  293  441  589  737  885
Maximum skips & repeats for a given input (so far):12

octave:10> date
ans = [redacted]
octave:11> clock
ans =

[redacted]

octave:12> diary off
```



TEST4B.M
WITH M = 2048
p = 2053

Pronounceable Passphrase Worksheet

by Edwin A. Suominen

Digit Content	Min. Digits	Entropy	Example Passphrase
Alternating consonants (C), Vowels (V) C,13: {b,d,g,h,k,l,m,n,p,r,s,t,z} V,5: {a,e,i,o,u}	C: 11 V: 10	About 64 bits, for minimum digits	nihudezo dogiz pozubume

The vowels and phonetically distinct consonanants below are pseudorandomly distributed, created using alternating pseudorandom lookups to a list of consonants b,d,g,h,k,l,m,n,p,r,s,t,z and vowels a,e,i,o,u. Alternating randomly selected consonants and randomly selected vowels from the array below form passphrases that have a faintly Oriental or African sound to them, and are more memorable than random alphanumerics. Consonants c,f,j,q,v,w,x,y are omitted because the passphrases tend to have a more distinct sound and are easier to pronounce without them.

You should split the consonant/vowel pairs into groups to make the passphrase pronounceable and thus more memorable. The suggested way of grouping the minimum 11 consonants and 10 vowels is as follows: CVCVCVCV CVCVC CVCVCVCV. Note that the middle group begins and ends with a consonant. The resulting passphrase has a distinct sound that makes you wonder if the "words" show up in some foreign language even though they're just groups of randomly chosen letters.

Unbend a paper clip slightly, repeatedly toss the clip onto a printout of this page without aiming it anywhere in particular, and select the consonant/vowel pair to which the unbent end comes closest to get the next digit in your consonant-vowel sequence. Don't use both digits from a pair – each digit in your passphrase needs its own toss. With good random tosses, you can expect the clip to bounce outside the array of digits below about half the time. Just toss again. Don't aim at any particular region.

ne	ze	nu	hi	ni	ba	hu	pi	pu	lu	me	ki	lu	ge	le	mu	nu	ko	se	ze	ta	ba	ga	ro	ta	be	ko	te
lo	ru	hu	gi	ra	ga	ro	do	ro	zu	tu	si	mu	pu	lo	bi	ze	tu	ha	ti	ru	to	ni	ze	bu	be	si	ma
ro	le	hi	ni	bo	ma	do	hu	ma	lu	bu	zi	ru	sa	ta	zo	po	go	ta	hi	ri	bi	te	la	me	go	ho	si
go	ka	hi	bo	li	ro	mi	ta	mo	ki	ku	hu	ri	le	da	ra	za	mi	le	mi	da	ra	zu	ki	ke	bi	mi	do
mo	ta	tu	ta	ra	si	la	go	ki	su	ki	mi	re	ba	le	so	to	za	ba	te	hi	ri	da	go	za	ka	sa	za
mo	na	la	me	ku	bu	mu	me	no	ke	ri	be	ho	tu	mu	ki	no	re	ho	pe	mi	hu	ho	pi	ge	ro	to	se
go	ka	ze	ri	du	sa	de	bi	si	de	mi	pi	se	po	le	mi	bu	re	pe	du	so	lu	la	ri	mo	mo	bo	go
to	ge	ne	go	lo	ru	sa	da	ma	ga	ni	pe	si	se	bu	di	pi	da	pu	ta	la	ku	hu	ko	go	za	si	de
tu	me	po	ro	to	pe	ku	hu	ra	ha	bu	zo	ti	na	ba	ma	se	te	pi	do	to	ke	so	do	re	ru	ze	ru
to	lo	ne	ki	ta	ha	go	go	li	ra	po	de	na	du	ba	su	po	ka	lu	bu	hi	be	za	la	ke	bo	ne	mi
he	ni	te	ma	ni	pu	ho	ru	ne	da	nu	de	ge	ge	so	ge	lu	re	ho	bi	po	li	ma	ri	su	pa	te	du
zi	ta	ma	lu	ti	bu	ha	ba	hu	mu	su	za	ko	to	ga	te	tu	ru	la	ki	ru	ze	po	ni	pi	ho	sa	pa
pa	bu	pi	za	du	go	ga	su	mu	re	su	ni	re	hi	bi	ko	po	la	zu	ri	ku	ka	zi	ba	pa	tu	di	da
ma	mo	di	hu	po	ro	ho	ku	se	pa	bo	la	ga	ne	me	pi	pu	gu	he	li	ha	gi	bo	he	hu	so	ru	pa
se	be	pe	ga	se	ni	lo	bo	se	ka	be	gi	de	zi	di	me	mu	lu	pe	ku	tu	go	pe	hi	di	bi	de	ne
mo	mi	ki	ni	ka	ha	nu	ka	za	do	he	ni	da	du	ri	se	du	lo	se	tu	pe	to	ti	hu	na	ru	se	ta
na	ho	bu	tu	ze	ge	ru	ru	ba	ga	li	ro	so	mu	du	he	da	go	mu	da	ne	tu	mu	pi	ne	bu	ka	la
li	le	he	me	gi	ru	ge	hi	lo	po	ho	pe	no	mu	pe	ta	lo	re	na	di	gi	ko	ze	re	pi	ga	ba	ki
ke	su	zo	ra	no	di	lu	du	be	za	na	po	do	su	ra	do	si	he	me	bi	ga	ra	ha	do	za	ru	ku	ri
zi	bu	do	lo	ga	ki	ha	zi	ka	ru	re	ke	ti	lo	zu	zu	gu	bu	ma	bo	hi	he	si	du	ne	hu	da	ka
te	ne	na	zi	hu	si	da	no	mo	te	bu	ko	tu	ro	mu	ne	ma	su	li	ba	ga	te	se	mo	bi	re	si	tu
ta	pe	bo	bu	ma	tu	ge	be	ri	no	pa	do	ri	za	ra	ho	so	to	da	ze	ze	lo	he	mi	ha	la	de	me
pu	ha	no	te	de	la	zi	hu	gi	ro	te	pa	mo	bu	mu	pe	da	sa	pe	na	pi	ti	ru	ro	pi	go	li	ku
gi	gi	pu	za	pe	go	ti	lo	pu	du	su	mo	ta	ki	ha	ge	ka	ke	pa	tu	po	hi	be	lu	sa	bo	li	so
ko	ra	ri	zi	po	ra	la	ze	pe	ru	zu	ra	mu	zu	du	ka	lo	ku	zo	po	mo	no	he	ze	su	ho	le	gi
te	go	pu	ka	ga	la	ki	lu	pa	ti	na	ga	gi	ba	ka	pi	ka	su	gi	mo	no	si	te	li	gi	se	zi	ze
ze	za	lu	ge	ro	zi	ra	do	bo	ma	zo	mi	ke	ze	la	bu	ru	be	ki	te	ga	ni	go	go	gi	ga	no	ha
si	bu	be	mi	ta	pa	de	ke	ta	lu	ru	ma	ra	si	se	te	me	pe	pu	gi	mu	me	li	bi	bi	hi	zu	me
de	he	ku	zu	nu	gi	pi	zi	ga	ka	ze	lo	re	ru	ha	zu	ta	ku	pi	po	me	la	da	mu	li	de	he	be
pi	le	mu	ko	ki	nu	ke	ga	hu	la	la	ri	po	su	ri	na	ru	no	ta	ro	ho	zi	so	ru	ri	go	su	hu
gi	ra	ru	pu	mu	gi	le	ga	ti	ne	go	le	ba	da	gi	si	si	ni	ra	zu	ma	ha	bu	le	se	ze	ro	go
pi	no	bu	ne	nu	ma	si	re	ka	gu	zo	si	bo	sa	be	zo	si	ko	po	li	gi	ra	pe	ne	gu	ri	la	ki
go	ka	ze	ne	ro	mo	bi	nu	he	no	hu	ne	ha	lo	ne	le	hu	hi	so	du	de	li	ta	ze	li	hu	de	do
ga	ha	bi	he	su	ku	pe	de	he	hu	ta	so	zo	he	bu	ti	hi	gi	ta	ti	ze	pe	gu	ni	ka	ra	su	pi
lu	ge	pe	ka	pu	na	da	mu	ti	du	mo	pi	de	pu	pa	ri	si	bi	sa	mi	li	ko	mi	be	ge	ku	su	po
me	mi	tu	ze	ge	ki	gu	hu	hi	lo	tu	da	mu	to	ru	gi	ru	ra	mo	na	ma	ba	si	se	ri	ro	ro	ka
po	ge	do	sa	pu	ma	so	na	ba	la	li	ka	ne	ba	di	te	zo	so	ru	mi	se	ni	gu	na	ta	di	zi	ri
ga	ga	ri	no	ho	sa	ne	mo	le	hi	he	re	ta	ne	go	ki	de	pi	de	gi	mu	du	tu	bo	to	ki	po	di
ma	ge	bo	se	ne	hi	ni	ma	go	li	le	ra	ri	mu	si	lo	gi	zo	ku	bo	ro	ze	po	to	do	go	re	tu
ga	ka	la	ki	za	ti	me	te	zi	ho	mi	ta	no	ne	lo	za	ma	mo	te	si	sa	ba	da	ku	ko	be	ki	go
zi	zu	tu	pe	ma	ba	zu	lo	ra	ge	hu	pa	te	ra	ra	ko	zu	ni	se	he	to	ma	se	gu	gu	ba	bi	te
pe	zo	ne	no	gu	ne	bi	ga	ra	na	ra	na	la	la	do	nu	du	za	ro	li	li	mu	ne	ru	ni	he	te	ma

Description	Digit Space #1	No. of #1 Digits	Entropy #1 (bits)	Digit Space #2	No. of #2 Digits	Entropy #2 (bits)	Digit Space #3	No. of #3 Digits	Entropy #3 (bits)	Total Entropy
6 word DICEWARE (http://diceware.com)	7776	6	77.54888	1	0	0	1	0	0	**77.5**
14 alphanumeric digits except lowercase "L" (can group 5-4-5 or 4-6-4)	35	14	71.80996	1	0	0	1	0	0	**71.8**
13 alphanumeric digits except lowercase "L" (can group 4-5-4)	35	13	66.68068	1	0	0	1	0	0	**66.7**
Pairs of phonetically distinct consonants "b,d,g,h,k,l,m,n,p,r,s,t,z") followed by vowel, arranged as follows: cvcvcvcv cvcvc cvcvcvcv	13	11	40.70484	5	10	23.21928	1	0	0	**63.9**
12 alphanumeric digits except lowercase "L" separated randomly into three groups	35	12	61.5514	11	1	3.459432	1	0	0	**65.0**
5 word DICEWARE	7776	5	64.62406	1	0	0	1	0	0	**64.6**
12 alphanumeric digits except lowercase "L" (can be grouped 4-4-4)	35	12	61.5514	1	0	0	1	0	0	**61.6**
4 words from 70K dictionary	70000	4	64.38027	1	0	0	1	0	0	**64.4**
Three dates, Month, Day, Year	12	3	10.75489	30	3	14.72067	500	3	26.89735	**52.4**
Random first name (10K) plus M.I. Plus last name (50K)	10000	1	13.28771	50000	1	15.60964	26	1	4.70044	**33.6**
Random street address	20000	1	14.28771	8	1	3	10000	1	13.28771	30.6
++	No.			NSEW		30.6	20	1	4.321928	**34.9**
(e.g.,	1	0	0	1	0	0	1	0	0	**0.0**
Passwords for smartcards)	1	0	0	1	0	0	1	0	0	**0.0**
Pairs of phonetically distinct consonants "b,d,g,h,k,l,m,n,p,r,s,t,z") followed by vowel, arranged as follows: cvcvcvcv cvcvc cvcvcvcv	13	4	14.80176	5	4	9.287712	1	0	0	**24.1**
5 alphanumeric digits except lowercase "L"	35	5	25.64642	1	0	0	1	0	0	**25.6**
(e.g.,	1	0	0	1	0	0	1	0	0	**0.0**
Medium Security for use with secure delay)	1	0	0	1	0	0	1	0	0	**0.0**
Pairs of phonetically distinct consonants "b,d,g,h,k,l,m,n,p,r,s,t,z") followed by vowel, arranged as follows: cvcvcvcv cvcvc cvcvcvcv	13	8	29.60352	5	8	18.57542	1	0	0	**48.2**
Pairs of phonetically distinct consonants "b,d,g,h,k,n,p,s,t,z") followed by vowel, arranged as follows: cvcvcvcv cvcvc cvcvcvcv	10	8	26.57542	5	8	18.57542	1	0	0	**45.2**
Groups of numbers 1000-9192 = {0-8192}+1000	8192	4	52	1	0	0	1	0	0	52
	1	0	0	1	0	0	1	0	0	**0.0**
	1	0	0	1	0	0	1	0	0	**0.0**

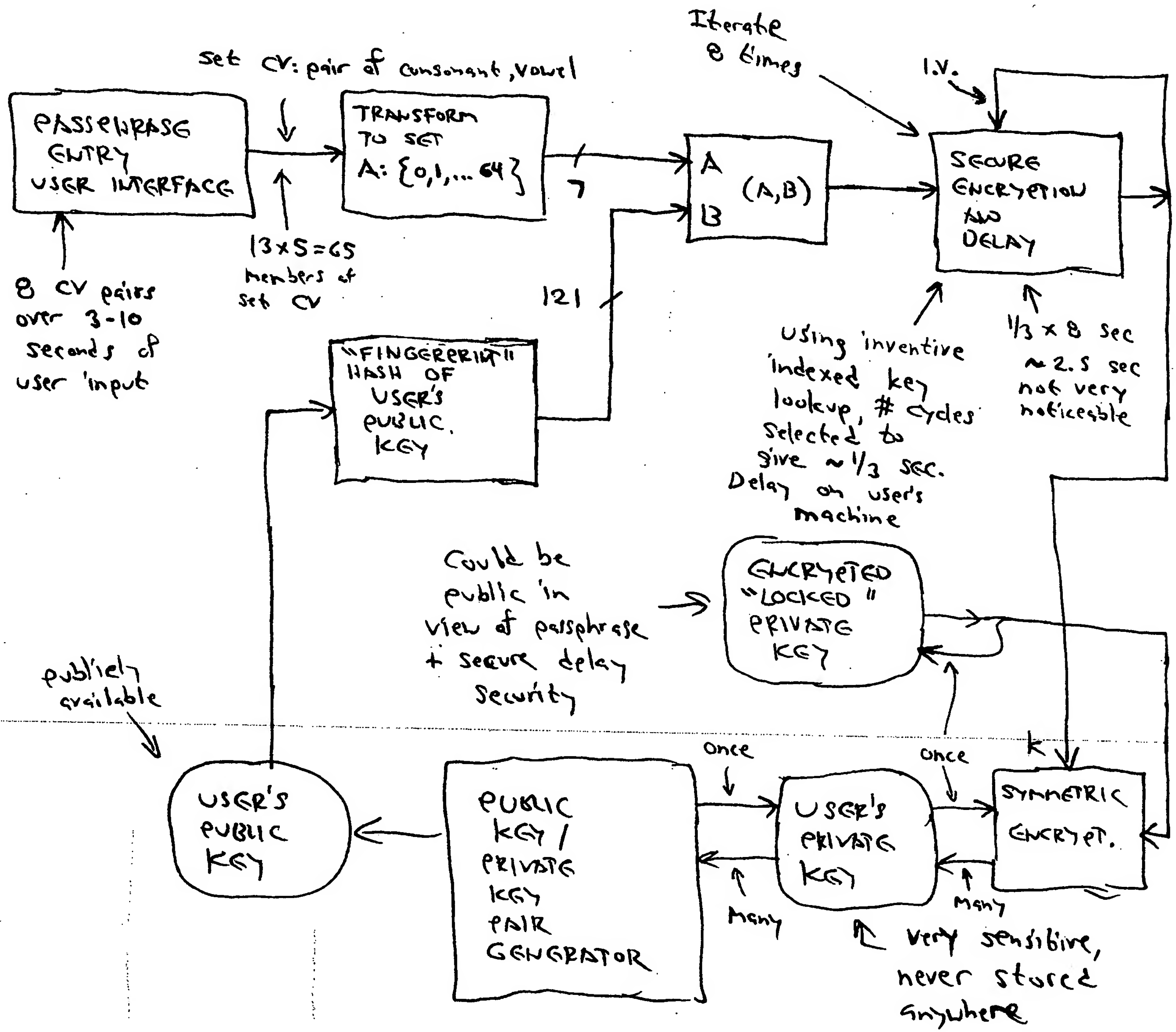
SECURE PASSPHRASE

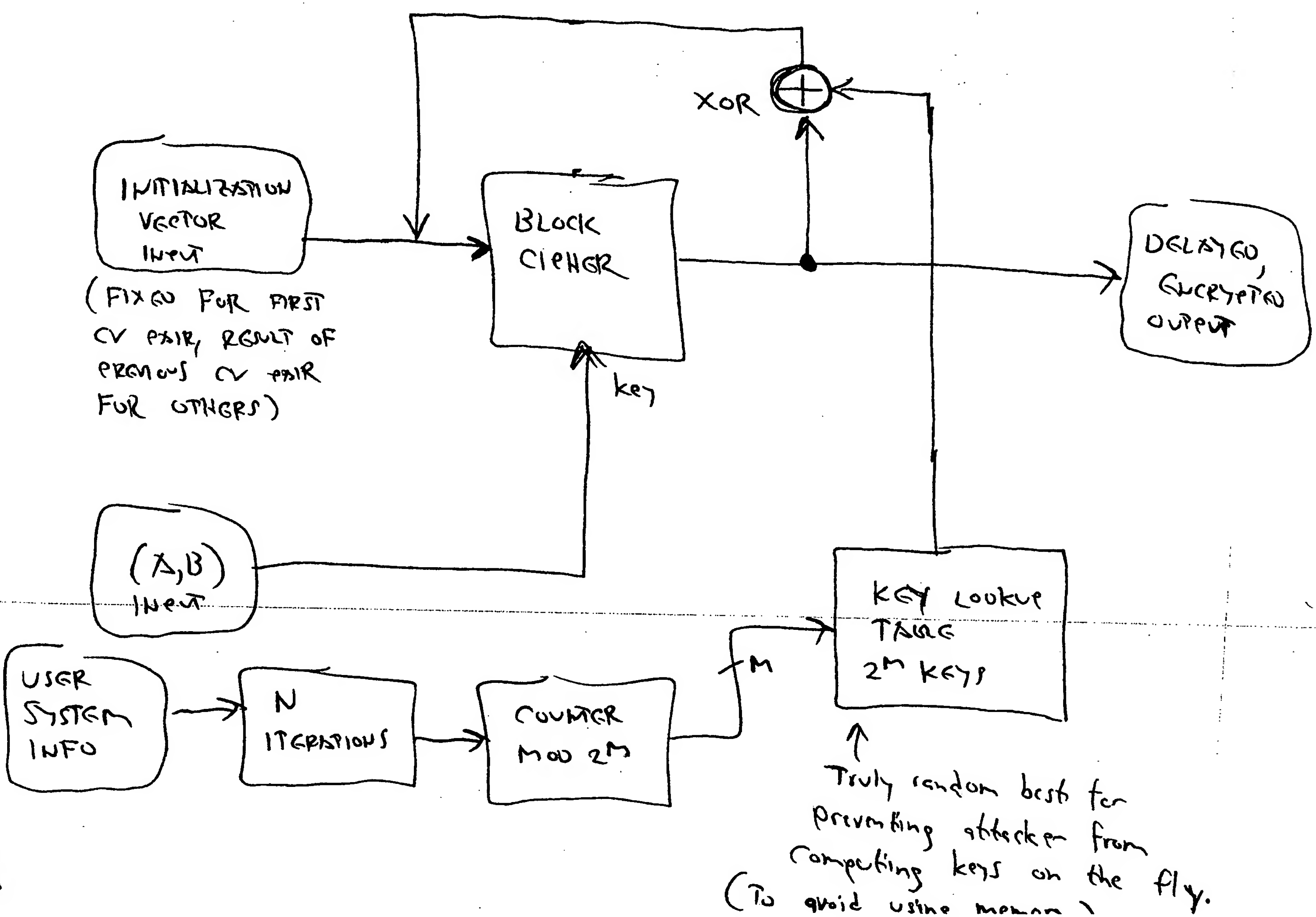
Z2-Z6 illustrate screen shots of a secure passphrase entry system according to various aspects of the invention, illustrating an exemplary user interface at different points during the input of a passphrase without the use of keystrokes. Thus, the security hazard of keystroke loggers can be avoided. In addition, the mouse-based input method may be preferred by users over the use of a keyboard, for example when they are entering their passphrase to browse encrypted e-mails or files. In an experiment the applicant carried out, "entering" the passphrase by the mouse input method (simulated by tapping a pen onto a printout similar to Z2-Z6) did not take him much longer than typing in the passphrase.

Advantageously, the passphrase is represented in the illustrated embodiment (as it is entered) both as circled letters and has a pair of stair-stepped line segments having characteristic shapes. Viewing the passphrase and its associated characteristic shapes of the line segments helps the user to remember the passphrase. Human brains are good at remembering pronounceable words (even when they are nonsense words) and are also good at remembering characteristic shapes. The combination of both characteristics of a unique passphrase can be expected to improve the user's ability to remember it when the time comes to input the passphrase.

A delay system according to another aspect of the invention, illustrated in the block diagrams of Z7 and Z8, makes a secure delay according to various aspects of the invention less unobtrusive to the user. It does so by beginning the delay process when the passphrase has been partially entered. Advantageously, such a system performs the delay computations substantially in parallel with the unavoidable delay of the user's input of the passphrase. Even when typing quickly, it took the applicant at least about three seconds to enter the passphrase during his experiment. This is a substantial period of delay that, when made computationally unavoidable, makes cracking the 2^48 possible combinations of the randomly chosen passphrase nearly impossible with the computing horsepower available around the date of filing of the present application. (See Z9 and Z10 for a detailed computational analysis.) The screen shots of Z2-Z6 show the "private key delayed unlocking" beginning with the first consonant-vowel pair entered by the user. The delayed unlocking (the inventive "computationally unavoidable" delay) continues substantially in parallel with the user's input of additional consonant-vowel pairs. Note Z6, in which the passphrase is confirmed and the private key has been completely unlocked.

# # #


set CV: pair of consonant, vowel
Iterate 8 times
I.V.
PASSPHRASE ENTRY USER INTERFACE
TRANSFORM TO SET A: {0,1,... 64}
7
A
B
(A,B)
SECURE ENCRYPTION AND DELAY
8 CV pairs over 3-10 seconds of user input
13×5=65 members of set CV
121
"FINGERPRINT" HASH OF USER'S PUBLIC KEY
Using inventive indexed key lookup, # cycles selected to give ~1/3 sec. Delay on user's machine
1/3 × 8 sec ~2.5 sec not very noticeable
Could be public in view of passphrase + secure delay security
ENCRYPTED "LOCKED" PRIVATE KEY
publicly available
USER'S PUBLIC KEY
PUBLIC KEY / PRIVATE KEY PAIR GENERATOR
Once
Many
USER'S PRIVATE KEY
Once
Many
K
SYMMETRIC ENCRYPT.
very sensitive, never stored anywhere


INITIALIZATION VECTOR INPUT
(FIXED FOR FIRST CV PAIR, RESULT OF PREVIOUS CV PAIR FOR OTHERS)
BLOCK CIPHER
XOR
key
DELAYED, ENCRYPTED OUTPUT
(A,B) INPUT
USER SYSTEM INFO
N ITERATIONS
COUNTER MOD 2M
M
KEY LOOKUP TABLE 2M KEYS
Truly random best for preventing attacker from computing keys on the fly.
(To avoid using memory)

1/2

Number of consonants	13	
Number of vowels	5	
Combinations in each CV pair	65	
Pairs	8	
Total Combinations	**318,644,812,890,625**	
Base-2 Entropy	48	(bits)

Mean Input Times (experiment)		
Touch typing (fast), hidden digits	4.70 (sec.)	=(5.3+ 4+ 5+ 5.1+ 4.1)/5
Tough typing (fast), digits shown	3.88	=(3.8+ 3.5+ 3.4+ 3.5+ 5.2)/5
Mouse, drag line through digits	9.32	= (9.7+ 9.2+ 9.8+ 8.9+ 9)/5
Mouse, click on digits	8.28	= (8+ 8.2+ 8.9+ 8+ 8.3)/5
Set total delay to minimum total input time (Keeps user from noticing the delay)	3.88 (sec.)	

Software (equivalent machine) Attack Analysis		
Total number of seconds for all delayed combinations (on equivalent machine)	1,236,341,874,015,620	
Average number of years on equivalent machine (1/2 total)	19,602,072	Impossible with present machines
Effective lifetime of signing key (years)	20	
Performance multiplier at end of life (Moore's law)	10,321	
Total number of seconds for all delayed combinations (on future machine)	119,785,790,491	
Number of future machines in network	1,000	Even this number would leave evidence of fraudulent activity on the part of the person forging signature with broken private key
Average number of years on future machine network (1/2 total)	**0.95**	

Massively Parallel Hardware (FPGA, ASIC) Attack Analysis		
Budget (current equivalent dollars)	1,000,000	
Cost per FPGA or ASIC (with NRE)	400	
Number of available parallel branches in budget	2,500	
Number of parallel branches operating simulateously	2,048	
Performance multiplier of each branch over equivalent machine	100	
Total performance multiplier over equivalent machine	204,800	
Total number of seconds for all delayed combinations	6,036,825,557	
Average number of years (1/2 total)	96	Impractical with present H/W, even w/ large budget
Effective lifetime of signing key (years)	20	

Can Include sunset date in ACI after which all sigs. are invalid

Performance multiplier at end of life (Moore's law)	10,321
Total number of seconds for all delayed combinations (on future hardware system)	584,892
Average number of days on future machine (1/2 total)	**3.38**
But, here's where the key lookup helps protect against such attacks...	
Random keys in key lookup table	8,192
Size of each key (in bytes)	16
Total memory for lookup table (bytes)	131,072
Total fast SRAM memory for all branches (bytes)	268,435,456
Total MB of fast SRAM memory	**262,144**
Cost per MB of SRAM (current equivalent dollars)	10
Total cost of SRAM	**2,621,440**
(See budget above.)	

(~~must~~ Should fit in 256K cache) to ensure top performance in user's machine. otherwise ratio between

$$\left[\frac{\text{Attacker Delay}}{\text{User Delay}}\right]$$

is reduced.

Lots of gates: $$

~ 8 × 2 gates per byte, or

~ 4.3 × 10⁹ gates

or ~ 2M gates per branch

Expensive ASIC!